Spacecraft Orbit Determination using the Filtering Estimator

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Abstract

This paper presents the study of the orbit determination of near-circular, low-Earth orbit (LEO) spacecraft using information of global position system (GPS) signals. The filtering estimator based on Kalman filter was implemented to estimate the position and velocity of the spacecraft. The orbit perturbation described by fourth term of zonal harmonics was included in the system model. The estimates were evaluated by the conventional orbit propagator.

Keywords: orbit determination, orbit perturbation, GPS, filtering estimator

1. Introduction

Since, the use of GPS has been successfully demonstrated in space navigation, most of space missions are now reply on the GPS measurements. However, the geometry of selected GPS satellites is one factor that magnifies the error in solution [1]. Error in GPS system and its measurements is another substantial factor that needs to be analysed [2].

The limitation of computational power of processing unit onboard satellite will lead to such an approach. The issues for space operation are still remaining in order to produce a compact flight-code and keep its accuracy. There are two alternative approaches that can be used to overcome the addressed limitations.

The first approach is to seek for some numerical integrator which can be used to propagate the orbit dynamic as fast and keep its accuracy. Recently, the state-of-art method namely, symplectic integration [3] has been proposed.

The second approach is to review the analytic formulation of orbit dynamics included the effect of perturbations, and to implement orbit estimator analytically [4], which requires no numerical integrator. However, it is necessary to consider the behaviour of long-term perturbation on satellite motion. As a system of satellite constellation and formation flying is a part of current and future trend in space program, this makes the analytic approach appears in many research activities in recent years [5, 6, 7].

In this paper, an extended Kalman filter was implemented to estimate the position and velocity of the spacecraft. The perturbed force caused by geo-potential was included in the system model. The simulated pseudoranges were used as the observable. Filtering initialisation was conducted by positioning derived from least square method [8].

2. Background

2.1 GPS Observable

The observable in GPS orbit determination is the pseudoranges (ρ) which means the apparent distance between tracked j^{th} GPS satellites and low-Earth orbit (LEO) spacecraft. The pseudorange can be obtained by

$$\rho_j = \sqrt{\left(x_j - x_u\right)^2 + \left(y_j - y_u\right)^2 + \left(z_j - z_u\right)^2} + ct_u \quad (1)$$

where (x_j, y_j, z_j) is a location of j^{th} GPS satellite,
(x_u, y_u, z_u) is a location of user LEO spacecraft,
t_u is an offset of receiver clock from the system time, and c is a speed of light.

2.2 Equations of Motion

The general form of equations of motion, including perturbations can be expressed as follows

$$\ddot{\mathbf{r}} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \mathbf{a}_{pr} \quad (2)$$

where $\mathbf{r}$ is the position vector of the satellite,
r is geocentric distance to satellite position,
$\mu_\oplus$ is the Earth-gravitational constant,
$\mathbf{a}_{pr}$ is the sum of the perturbing accelerations.

If the term of perturbing accelerations is ignored, the equation is known as the case of the 2-body problem.

2.1 Geo-potential Function

The geo-potential, U, at any point specified by the spherical coordinates $(r, \phi_{sat}, \lambda_{sat})$ can be expressed in form of spherical harmonics [9]

$$U(r,\phi_{sat},\lambda_{sat}) = \frac{\mu_\oplus}{r}\left[1 - \sum_{n=2}^{\infty} J_n\left(\frac{R_\oplus}{r}\right)^n P_n(\sin\phi_{sat}) + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_\oplus}{r}\right)^n P_{n,m}(\sin\phi_{sat})\cos m(\lambda_{sat} - \lambda_{n,m})\right] \quad (3)$$

where $R_\oplus$ is a mean equatorial radius of the Earth, P_n is Legendre polynomial of degree n and zero order, P_{nm} is associated Legendre polynomial of degree n and order *m*, *J* is a coefficient of harmonics, ϕ_{sat} is geocentric latitude, and λ_{sat} is geocentric longitude.

The geo-potential function expressed in form of the zonal harmonics is given in Table 1

Table 1 Geo-potential function

order *n*	geo-potential function U_{J_n}
(2)	$U_{J_2} = -\frac{J_2}{2}\frac{\mu_\oplus}{r}\left(\frac{R_\oplus}{r}\right)^2 (3\sin^2\phi_{sat} - 1)$
(3)	$U_{J_3} = -\frac{J_3}{2}\frac{\mu_\oplus}{r}\left(\frac{R_\oplus}{r}\right)^3 \left(5\sin^3\phi_{sat} - 3\sin\phi_{sat}\right)$
(4)	$U_{J_4} = -\frac{J_4}{8}\frac{\mu_\oplus}{r}\left(\frac{R_\oplus}{r}\right)^4 \left(35\sin^4\phi_{sat} - 30\sin^2\phi_{sat} + 3\right)$

The perturbing accelerations caused by geo-potential, $\mathbf{a}_{Earth\_potential}$, can be found from

$$\mathbf{a}_{Earth\_potential} = \nabla U = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k} \quad (4)$$

where ∇ denotes gradient operation

3. Implemented Filtering Estimator

Filtering estimator based on extended Kalman filter (EKF) is implemented to estimate the positioning and velocity of the artificial spacecraft. The assumptions of the EKF estimator were addressed in [8].

3.1 State Vector

The state vectors **x** is defined as

$$\mathbf{x} = \begin{bmatrix}\mathbf{r} & \mathbf{v} & \mathbf{b}\end{bmatrix}^T \quad (5)$$

where $\mathbf{r} = \begin{bmatrix}x_u & y_u & z_u\end{bmatrix}^T$ is a position vector of user,

$\mathbf{v} = \begin{bmatrix}v_x & v_y & v_z\end{bmatrix}^T$ is a velocity vector of user,

$\mathbf{b} = \begin{bmatrix}b & \dot{b}\end{bmatrix}^T$ consists of clock offset and its rate.

3.2 System Model

The non-linear model is defined as [11]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{w}(t) \quad (6)$$

where $\mathbf{f}(\mathbf{x}, t)$ is a non-linear system model, $\mathbf{w}(t)$ is a zero mean white system noise with covariance matrix **Q**

The error between the actual state vector, $\mathbf{x}$, and estimate, $\hat{\mathbf{x}}$, is defined as the state perturbation, $\Delta\mathbf{x}$

$$\Delta\mathbf{x}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \quad (7)$$

As it is assumed that $\Delta\mathbf{x}$ is small, the system model can be approximately derived from

$$\mathbf{f}(\mathbf{x}, t) \approx \mathbf{f}(\hat{\mathbf{x}}, t) + \mathbf{F}\cdot\Delta\mathbf{x} \quad (8)$$

where **F** is a linearised system model defined as

$$\mathbf{F} = \left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right] = \frac{\partial(\dot{\mathbf{r}} \;\; \dot{\mathbf{v}} \;\; \dot{\mathbf{b}})}{\partial(\mathbf{r} \;\; \mathbf{v} \;\; \mathbf{b})} \quad (9)$$

As we consider the effect of perturbations, the dynamics equation of the system model is expressed as

$$\dot{\mathbf{x}} = \begin{bmatrix}\dot{\mathbf{r}} & \dot{\mathbf{v}} & \dot{\mathbf{b}}\end{bmatrix}^T + \mathbf{w}(t) \quad (10)$$

where $\dot{\mathbf{v}} \equiv \ddot{\mathbf{r}} = -\mu\frac{\mathbf{r}}{r^3} + \mathbf{a}_{pr}$, $\dot{\mathbf{b}} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\mathbf{b}$,

The state transition matrices, **Φ**, can be approximated for a short sampling period Δt

$$\mathbf{\Phi} \approx \mathbf{I}_{8\times8} + \mathbf{F}\Delta t \quad (11)$$

where $\Delta t = t_{(p+1)} - t_{(p)}$,

The discrete state perturbation model is given by

$$\Delta\mathbf{x}_{(p+1)} = \mathbf{\Phi}_{(p)}\Delta\mathbf{x}_{(p)} \quad (12)$$

3.3 Measurement model

A discrete non-linear measurement model is

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, t) + \mathbf{m}(t) \quad (13)$$

where $\mathbf{m}(t)$ is a zero mean white measurement noise with scalar covariance *R*, and $\mathbf{h}(\mathbf{x},t)$ is a non-linear output

$$\mathbf{h}(\mathbf{x}, t) = \begin{bmatrix}\rho_1 & \rho_2 & \cdots & \rho_j\end{bmatrix}^T \quad (14)$$

At epoch *p*, the estimated pseudoranges is computed from (2). The innovation, $\delta\rho$, is computed as the scalar difference between estimated pseudoranges $\hat{\rho}$ and measured pseudoranges, ρ. For all GPS data, the innovations are stacked into a vector $\Delta\boldsymbol{\rho}_{(p)}$.

The linearised innovation error model is given by

$$\Delta\boldsymbol{\rho}_{(p)} = \mathbf{z}_{(p)} - \mathbf{h}_{(p)}(\hat{\mathbf{x}},\; t) = \mathbf{H}_{(p)}\Delta\mathbf{x}_{(p)} + \mathbf{m}_{(p)}(t) \quad (15)$$

where $\Delta\mathbf{r}_{(p)}$ is an innovation vector at epoch *p*, an observation matrix $\mathbf{H}_{(p)}$ is defined as

$$\mathbf{H}_{(p)} = \left[\frac{\partial\mathbf{h}_{(p)}}{\partial\mathbf{x}}\right]_{\mathbf{x}=\hat{\mathbf{x}}} \quad (16)$$

3.4 Positioning Acquisition

Given an initial values of approximate components of positioning $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ and offset clock $\hat{t}_u$, the estimated pseudoranges is then obtained from

$$\hat{\rho}_j = \sqrt{\left(x_j - \hat{x}_u\right)^2 + \left(y_j - \hat{y}_u\right)^2 + \left(z_j - \hat{z}_u\right)^2} + c\hat{t}_u \quad (17)$$

The difference between the measured pseudorange and estimated pseudoranges is expressed as

$$\Delta\rho_j = \hat{\rho}_j - \rho_j \quad (18)$$

In single time frame called epoch, it is assumed that *n* GPS satellites are tracked, therefore, a set of the $\Delta\rho_j$ can be formed in the vector of measurement error, $\Delta\boldsymbol{\rho}$,

$$\Delta\boldsymbol{\rho} = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \end{bmatrix} = \begin{bmatrix} \hat{\rho}_1 - \rho_1 \\ \hat{\rho}_2 - \rho_2 \\ \vdots \\ \hat{\rho}_n - \rho_n \end{bmatrix} \quad (19)$$

A normalised vector which lies on the direction of the approximate positioning $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ as points to the tracked j^{th} GPS satellite (x_j, y_j, z_j), can be expressed as

$$\mathbf{d}_j = \left[\left(\frac{x_j - \hat{x}_u}{\hat{r}_j}\right) \quad \left(\frac{y_j - \hat{y}_u}{\hat{r}_j}\right) \quad \left(\frac{z_j - \hat{z}_u}{\hat{r}_j}\right)\right]^T \quad (20)$$

where

$$\hat{r}_j = \sqrt{\left(x_j - \hat{x}_u\right)^2 + \left(y_j - \hat{y}_u\right)^2 + \left(z_j - \hat{z}_u\right)^2} \quad (21)$$

As it is assumed that *n* GPS satellites are tracked, therefore, a set of normalised vectors can be formed in the observation matrix **G**

$$\mathbf{G} = \begin{bmatrix} \mathbf{d}_1 & 1 \\ \mathbf{d}_2 & 1 \\ \vdots & \vdots \\ \mathbf{d}_n & 1 \end{bmatrix} \quad (22)$$

The goodness of fit in least squares sense is given by

$$\Delta\mathbf{g} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\Delta\boldsymbol{\rho} \quad (23)$$

where $\Delta\mathbf{g}$ is an error vector of positioning and receiver clock offset

$$\Delta\mathbf{g} = \left[\Delta x_u \quad \Delta y_u \quad \Delta z_u \quad -c\Delta t_u\right]^T \quad (24)$$

An trail of approximate components for iterative process is given by

$$\begin{aligned} \hat{x}_{u(p+1)} &= \hat{x}_{u(p)} + \Delta\hat{x}_{u(p)} \\ \hat{y}_{u(p+1)} &= \hat{y}_{u(p)} + \Delta\hat{y}_{u(p)} \\ \hat{z}_{u(p+1)} &= \hat{z}_{u(p)} + \Delta\hat{z}_{u(p)} \\ \hat{t}_{u(p+1)} &= \hat{t}_{u(p)} + \Delta\hat{t}_{u(p)} \end{aligned} \quad (25)$$

where *p* denotes epoch.

As presented in Equation 8, if the measurement error is minimised (another word, the estimated pseudoranges are very close to the measured pseudoranges), therefore the error in solution will be came down to satisfy the desired figure.

4. Test Results

Simulation results presented in this paper are based on a three-axis stabilised satellite in a circular orbit, 64.5 degrees inclination, and altitude 650 km. The spacecraft orbit was propagated using SGP4 (Simplified General Perturbations 4) orbit propagator [12], which includes geo-gravitational and drag models. The orbits of GPS constellation were propagated using SDP4 (Simplified Deep Perturbations 4), which used for deep-space satellite. The nominal simulation parameters are given in Table 2. An error budget of pseudoranges is given in [8].

Table 2 Simulation Parameters for LEO satellite

	parameter	value	unit
nominal orbit	semi-major axis	7028	km
	eccentricity	0.142	-
	inclination	64.5	deg
physical structure	height	1200	mm
	diameter	1100	mm
	weight	300	kg
moment of inertia	X axis	40.45	(kg-m^2)
	Y axis	42.09	(kg-m^2)
	Z axis	40.36	(kg-m^2)

In this paper, the measurement error is assumed as white Gaussian with 10 metres rms [8]. The NORAD 2-line elements of UoSat12 and operational GPS satellites are used as the initial figures for orbit propagations.

One week of simulated GPS measurements (10 second interval) are used as the input file. The setup parameters for the implemented EKF estimator are shown in Table 3.

Table 3 Setup parameters for EKF estimator

Parameter	value	dimension
system noise variance, **Q**	25	mixed
measurement noise variance, R	100	$metre^2$

4.1 Acquisition Phase

As shown in Figure 1, the point estimation in acquisition phase was performed over a given period, for instance the first 2 minutes, by using the least squares method. The large disparity in the 2^{nd} minute caused by the initialisation of the filtering estimator.

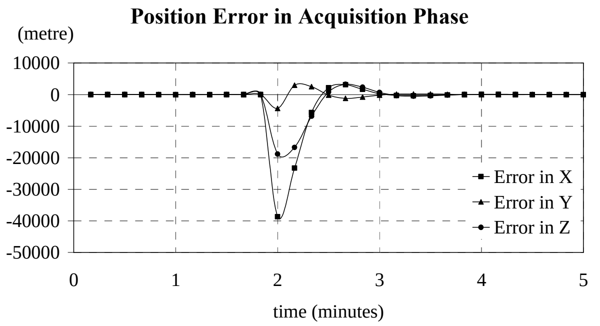

Figure 1. Position disparity in acquisition phase

4.2 Continuing Phase

Once the filtering estimator was initialised, the point solution of positioning and velocity is fully estimated, and compared to the reference solutions provided by SGP4. The difference of positioning between GPS solution and SGP4 was shown in Figure 2. The difference of velocity between GPS solution and SGP4 was shown in Figure 3.

It can be seen that the disparity in positioning was within 20 metres approximately, whereas the disparity in velocity was within 2 metre per second.

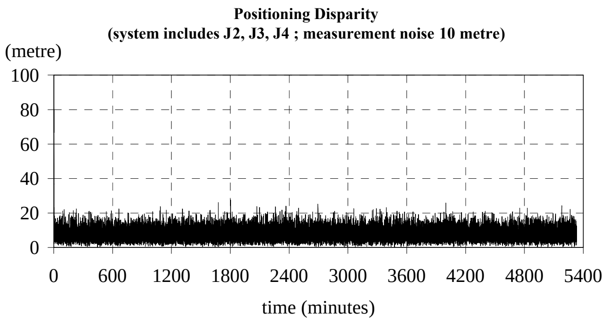

Figure 2. Positioning disparity

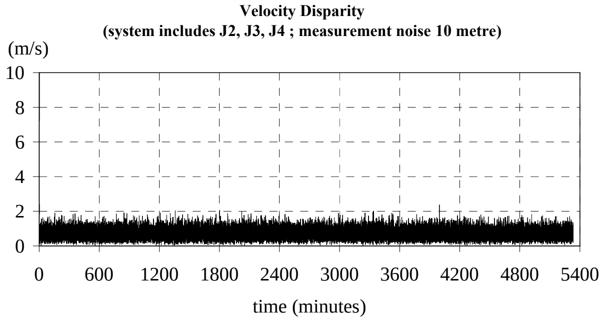

Figure 3. Velocity disparity

The computed one-sigma rms of disparity between the estimates and SGP4 was shown in Table 5.

Table 5 One-sigma rms of disparity between the SGP4's solutions and estimates (system includes J_2, J_3, J_4)

positioning	disparity (1σ)	velocity	disparity (1σ)
X axis	5.22 m	X axis	0.47 m/s
Y axis	4.99 m	Y axis	0.42 m/s
Z axis	5.39 m	Z axis	0.46 m/s

DOP (dilution of precision) parameters can be used to expresses the effect of geometry of selected GPS satellites to the estimated positioning. From Equation (23), the geometric DOP (GDOP) parameters are computed by

$$\mathrm{GDOP} = \sqrt{tr\left\{(\mathbf{G}^T\mathbf{G})^{-1}\right\}\Big|_{4\times 4}} \qquad (26)$$

where tr denotes trace of matrix.

The computed GDOP (geometric DOP) is shown in Figure 4.

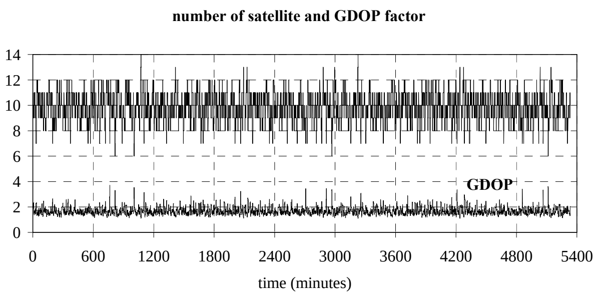

Figure 4. Computed GDOP

As shown in Figure 4, it can be seen that the GDOP figure went higher when the number of satellite dropped to six satellites. This explained that the number of tracked satellites is one factor taken into account of DOP calculation.

5. Conclusions

The filtering estimator was implemented to estimate the orbit of LEO artificial spacecraft from code information of GPS signals. The filtering initialisation was making use of the conventional least squares to provide the initial guess. The positioning and velocity were estimated from the dynamic model of filtering estimator. The forth order of zonal harmonics of the Earth's gravitation fields was taken into account. The simulated results provided the substantial information for further study.

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