Statistical Orbit Determination for Low-Earth-Orbit Spacecraft Using Code Information of Global Position System (GPS) Signals

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Abstract

This paper presents the study of the orbit determination of near-circular, low-Earth orbit (LEO) spacecraft using code information of global position system (GPS) signals. The statistical estimator based on linearlised least squares was implemented to estimate the position and velocity of the spacecraft. The two-body motion was included in the variational model. The advantage of statistical estimator over the conventional least square estimation is that the positioning and velocity of user was estimated using the single-type observations. The estimates were evaluated by the conventional orbit propagator.

Keywords: statistical orbit determination, GPS, differential correction, least squares

1. Introduction

There are several factors influenced in precision of satellite orbit determination, such as satellite tracking techniques and its observations, computational methods, and model of orbit perturbations.

Since, the use of GPS has been successfully demonstrated in space navigation [1], most of space missions are now reply on the GPS measurements. However, the geometry of selected GPS satellites is one factor that magnifies the error in solution [2]. Error in GPS system and its measurements is another substantial factor that needs to be analysed [3].

The limitation of computational power of processing unit onboard satellite will lead to such an approach. The issues for space operation are still remaining in order to produce a compact flight-code and keep its accuracy. There are two alternative approaches that can be used to overcome the addressed limitations.

The first approach is to seek for some numerical integrator which can be used to propagate the orbit dynamic as fast and keep its accuracy. Recently, the state-of-art method namely, symplectic integration [4] has been proposed. The second approach is to review the analytic formulation of orbit dynamics included the effect of perturbations, and to implement orbit estimator analytically [5], which requires no numerical integrator. However, it is necessary to consider the behaviour of long-term perturbation on satellite motion. As a system of satellite constellation and formation flying is a part of current and future trend in space program, this makes the analytic approach appears in many research activities in recent years [6, 7, 8].

The filtering estimator such as extended Kalman filter is another computational method which can be used to estimate the position and velocity of the spacecraft. However, the filtering estimator require initial guess for initialisation which may require other methods [9].

The differential correction technique associated with least squares estimation has been used to find the position and velocity of TOPEX [10] from dense observation data.

This paper present the statistical approach based on least squares was implemented to estimate the position and velocity of the spacecraft. The two-body motion was restricted in the variational model. The advantage of statistical estimator over the conventional least square estimation is that the positioning and velocity was estimated from a single-type observation.

2. Background

2.1 GPS Observable

The observable in GPS orbit determination is the pseudoranges (ρ) which means the apparent distance between tracked j^{th} GPS satellites LEO spacecraft. The pseudorange can be obtained by

$$\rho_j = \sqrt{\left(x_j - x_u\right)^2 + \left(y_j - y_u\right)^2 + \left(z_j - z_u\right)^2} + ct_u \quad (1)$$

where (x_j, y_j, z_j) is a location of j^{th} GPS satellite,
(x_u, y_u, z_u) is a location of user LEO spacecraft,
t_u is an offset of receiver clock from the system time, and c is a speed of light.

Given an initial values of approximate components of positioning $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ and offset clock $\hat{t}_u$, the estimated pseudoranges is then obtained from

$$\hat{\rho}_j = \sqrt{\left(x_j - \hat{x}_u\right)^2 + \left(y_j - \hat{y}_u\right)^2 + \left(z_j - \hat{z}_u\right)^2} + c\hat{t}_u \quad (2)$$

The difference between the measured pseudorange and estimated pseudoranges is expressed as

$$\Delta\rho_j = \hat{\rho}_j - \rho_j \tag{3}$$

In single time frame, it is assumed that n GPS satellites are tracked, therefore, a set of the $\Delta\rho_j$ can be formed in the vector of measurement error, $\Delta\boldsymbol{\rho}$,

$$\Delta\boldsymbol{\rho} = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \end{bmatrix} = \begin{bmatrix} \hat{\rho}_1 - \rho_1 \\ \hat{\rho}_2 - \rho_2 \\ \vdots \\ \hat{\rho}_n - \rho_n \end{bmatrix} \tag{4}$$

2.2 Equations of Motion

The general form of equations of motion, including perturbations can be expressed as follows

$$\ddot{\mathbf{r}} = -\frac{\mu_\oplus}{r^3}\mathbf{r} + \mathbf{a}_{pr} \tag{5}$$

where $\mathbf{r}$ is the position vector of the satellite,
r is geocentric distance to satellite position,
$\mu_\oplus$ is the Earth-gravitational constant,
$\mathbf{a}_{pr}$ is the sum of the perturbing accelerations.

If the term of perturbing accelerations is ignored, the equation is known as the case of the 2-body motion.

2.3 Differential Correction Technique Associated With Least Squares Estimation

The differential correction technique is a tool which can be used to estimate an orbit's state from measurement of the satellite motion.

The differential-correction equation for over-determine least squares estimation is given by

$$\boldsymbol{\delta}\mathbf{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\boldsymbol{\delta\rho} \tag{6}$$

where $\boldsymbol{\delta}\mathbf{x}$ is a corrections to the state vector,
$\mathbf{A}$ is a partial-derivative matrix,
$\boldsymbol{\delta\rho}$ is residual observables,
and ^{T} denotes transpose of matrix

To formulate the **A** matrix, other partial-derivative matrices, **F** and **Φ**, are required. The **F** matrix is a critical and complex component of the overall **A** matrix. The **F** matrix primary use is to find the state transition matrix (**Φ**).

Considering the beginning of batch at t_o and an given epoch at t, we can find a relationship between **F** and **Φ** from a first-order differential equation

$$\dot{\boldsymbol{\Phi}}(t,t_o) = \mathbf{F}(t)\boldsymbol{\Phi}(t,t_o) \tag{7}$$

where **F** matrix is partial derivative of state vector function, $f(\mathbf{x})$, with respect to the state vector $\mathbf{x}$ [10].

$$\mathbf{F} = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \tag{8}$$

3. Implemented Estimator

The statistical estimator based on least squares is implemented to estimate the positioning and velocity of the artificial spacecraft.

3.1 State Vector

The state vectors $\mathbf{x}$ is defined as

$$\mathbf{x} = \begin{bmatrix} \mathbf{r} & \mathbf{v} & t_u \end{bmatrix}^T \tag{9}$$

where $\mathbf{r} = \begin{bmatrix} x_u & y_u & z_u \end{bmatrix}^T$ is a position vector of user,
$\mathbf{v} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$ is a velocity vector of user,
t_u is an offset of receiver clock from the system time.

3.2 Force Model

The force model assumed for the restricted 2-body excludes the perturbations

$$\dot{\mathbf{r}} = \mathbf{v} \quad ; \quad \dot{\mathbf{v}} = \mathbf{a} \tag{10}$$

where $\mathbf{a} = -\frac{\mu_\oplus}{r^3}\mathbf{r}$

3.3 Partial Derivative Matrices

Let denotes $\mathbf{x}_o$ is the state at the beginning of the batch measurements. The observation matrix $\mathbf{A}_t$ at a given time t, is assumed to be the function of the current state as

$$\mathbf{A}_t = \frac{\partial \boldsymbol{\rho}_t}{\partial \mathbf{x}_o} = \frac{\partial \boldsymbol{\rho}_t}{\partial \mathbf{x}_t}\frac{\partial \mathbf{x}_t}{\partial \mathbf{x}_o} \tag{11}$$

The above equation distinguishes the observation partial derivative $(\partial\boldsymbol{\rho}_t / \partial\mathbf{x}_t)$ from the partial derivative of the state over time. The term of $(\partial\mathbf{x}_t / \partial\mathbf{x}_o)$ is noticed as the state transition matrix, **Φ**.

The state transition matrix, **Φ**, is given by [11]

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{rv} & 0 \\ 0 & 1 \end{bmatrix} \tag{12}$$

where $\boldsymbol{\Phi}_{rv}$ denotes the state transition matrix (6×6 dimension) for orbital parameters

Then we can find the $\mathbf{\Phi}_{rv}$ by solving the set of 36 couples equations

$$\dot{\mathbf{\Phi}}_{rv} = \mathbf{F}\mathbf{\Phi}_{rv} \tag{13}$$

where

$$\mathbf{F} = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \frac{\partial f(\mathbf{r}, \mathbf{v}, t_u)}{\partial(\mathbf{r}, \mathbf{v}, t_u)} \tag{14}$$

The **F** matrix can be written in the block form as four (3×3) matrices

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \tag{15}$$

where

$$\mathbf{F}_{11} = \frac{\partial \dot{\mathbf{r}}}{\partial \mathbf{r}} = \mathrm{O} \quad ; \quad \mathbf{F}_{12} = \frac{\partial \dot{\mathbf{r}}}{\partial \mathbf{v}} = \mathbf{I} \tag{16}$$

$$\mathbf{F}_{21} = \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}} \quad ; \quad \mathbf{F}_{22} = \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{v}} = \mathrm{O} \tag{17}$$

O denotes the zero matrix,
and **I** denotes the identity matrix

Once we substitute equation (15) into equation (13) then we can partition the state transition matrix into four (3×3) matrices

$$\mathbf{\Phi}_{rv} = \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \end{bmatrix} \tag{18}$$

Where

$$\dot{\mathbf{\Phi}}_{rv} = \begin{bmatrix} \dot{\mathbf{\Phi}}_{11} & \dot{\mathbf{\Phi}}_{12} \\ \dot{\mathbf{\Phi}}_{21} & \dot{\mathbf{\Phi}}_{22} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \\ \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}}\mathbf{\Phi}_{11} & \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}}\mathbf{\Phi}_{12} \end{bmatrix} \tag{19}$$

The correction vector which computed from equation (6) can be used to update the state vector (**x**) by

$$\mathbf{x}_t = \mathbf{x}_o + \boldsymbol{\delta}\mathbf{x}_o \tag{20}$$

4. Test Results

Simulation results presented in this paper are based on a three-axis stabilised satellite in a circular orbit, 64.5 degrees inclination, and altitude 650 km. The spacecraft orbit was propagated using SGP4 (Simplified General Perturbations 4) orbit propagator [12], which includes geo-gravitational and drag models. The orbits of GPS constellation were propagated using SDP4 (Simplified Deep Perturbations 4), which used for deep-space satellite. The nominal simulation parameters are given in Table 1.

Table 1 Simulation Parameters for LEO satellite

	parameter	value	unit
Nominal Orbit	semi-major axis	7028	km
	eccentricity	0.142	-
	inclination	64.5	deg
Physical Structure	height	1200	mm
	diameter	1100	mm
	weight	300	kg
moment of Inertia	X axis	40.45	(kg-m^2)
	Y axis	42.09	(kg-m^2)
	Z axis	40.36	(kg-m^2)

In this paper, the measurement error is assumed as white Gaussian with 10 metres rms [9]. The NORAD 2-line elements of UoSat12 and operational GPS satellites are used as the initial figures for orbit propagations.

The five days of simulated GPS measurements (10 second interval) are used as the input file. Once the statistical estimator was initialised, the point solution of positioning and velocity is estimated, and compared to the reference solutions provided by SGP4. The difference of positioning between GPS solution and SGP4 was shown in Figure 1. The difference of velocity between GPS solution and SGP4 was shown in Figure 2.

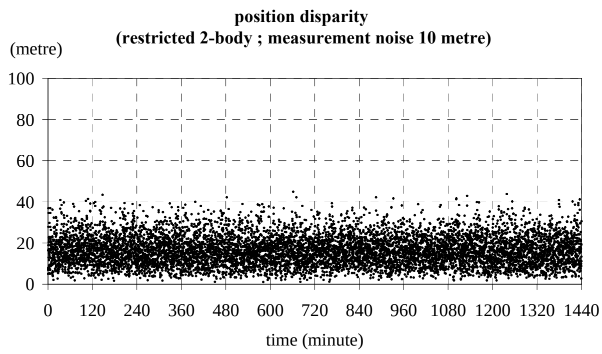

Figure 1. Position disparity

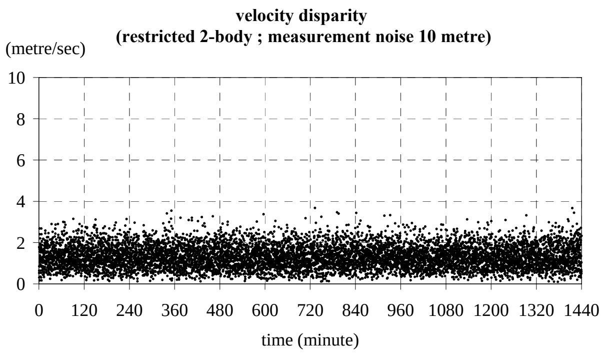

Figure 2. Velocity disparity

It can be seen that the disparity in positioning was within 40 metres approximately, whereas the disparity in velocity was within 3 metre per second.

The computed one-sigma rms of disparity between the estimates and SGP4 was shown in Table 2.

Table 2 One-sigma rms of disparity between the SGP4's solutions and estimates

positioning	disparity (1σ)	velocity	disparity (1σ)
X axis	10.46 m	X axis	0.84 m/s
Y axis	10.12 m	Y axis	0.81 m/s
Z axis	10.47 m	Z axis	0.82 m/s

DOP (dilution of precision) parameters can be used to expresses the effect of geometry of selected GPS satellites to the estimated positioning. From Equation (6), the geometric DOP (GDOP) parameters are computed by

$$\mathrm{GDOP} = \sqrt{tr\left\{(\mathbf{A}^T\mathbf{A})^{-1}\right\}\Big|_{4\times4}} \qquad (21)$$

where *tr* denotes trace of matrix.

The computed GDOP (geometric DOP) is shown in Figure 3.

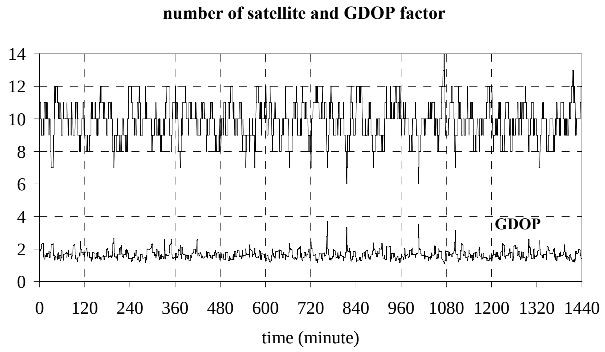

Figure 3 Computed GDOP

As shown in Figure 3, it can be seen that the GDOP figure went higher when the number of satellite dropped to six satellites. This explained that the number of tracked satellites is one factor taken into account of DOP calculation.

5. Conclusions

The statistical estimator based on least squares was implemented to estimate the orbit of LEO artificial spacecraft from code information of GPS signals. The differential correction was exploited to find the partial derivative matrices for restricted 2 body motion. The positioning and velocity were estimated from the estimator. The simulated results provided the substantial information for further study.

6. Ongoing Work

The further study will focus on orbit perturbations. The Earth's gravitational fields will be included in the motion equation. The secular effect of atmosphere drag is another one concerning that we may need to take into account.

7. Acknowledgement

An author would like to thank Dr.Yoshi Hashida for his constructive advices through out the study.

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